Control midterm q3

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Consider the system described by

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}\mathbf{x}$$

Obtain the response of the system for $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $u(t)=0$.

$$[A] = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, [B] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, [C] = \begin{bmatrix} 0 & 1 \end{bmatrix}, [D] = 0$$

$$\phi_{(s)} = [sI - A]^{-1}$$

$$[sI - A] = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} s-2 & 1 \\ -4 & s-3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{\underbrace{(s-2)(s-3)+4}_{\underbrace{s^2-5s+6+4}_{s^2-5s+10}}} \begin{bmatrix} s-2 & 1 \\ -4 & s-3 \end{bmatrix}$$

$$\Rightarrow s_{1,2} = \frac{5 \pm \sqrt{5^2 - 4\times1\times10}}{2\times1}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s-2}{s^2-5s+10} & \frac{1}{s^2-5s+10} \\ -\frac{4}{s^2-5s+10} & \frac{s-3}{s^2-5s+10} \end{bmatrix}$$

$$\Bigg[\frac{(s-2.5)+0.5}{} \qquad \frac{1}{} \times \frac{\sqrt{3.75}}{} \Bigg]$$

$$= \begin{bmatrix} \dfrac{(s-2.5)+0.5}{(s-2.5)^2+3.75} & \dfrac{1}{\sqrt{3.75}} \times \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} \\ \\ -\dfrac{4}{\sqrt{3.75}} \times \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} & \dfrac{(s-2.5)-0.5}{(s-2.5)^2+3.75} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s-2.5}{(s-2.5)^2+3.75} + \dfrac{0.5}{\sqrt{3.75}} \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} & \dfrac{1}{\sqrt{3.75}} \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} \\ \\ -\dfrac{4}{\sqrt{3.75}} \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} & \dfrac{s-2.5}{(s-2.5)^2+3.75} - \dfrac{0.5}{\sqrt{3.75}} \dfrac{\sqrt{3.75}}{(s-2.5)^2+3.75} \end{bmatrix}$$

$$\Phi_{(t)} = \mathcal{L}^{-1}\left\{\Phi_{(s)}\right\} \Rightarrow$$

$$\Phi_{(t)} = \begin{bmatrix} e^{2.5t}\cos\sqrt{3.75}\,t + \dfrac{0.5}{\sqrt{3.75}} e^{2.5t}\sin\sqrt{3.75}\,t & \dfrac{1}{\sqrt{3.75}} e^{2.5t}\sin\sqrt{3.75}\,t \\ \\ -\dfrac{4}{\sqrt{3.75}} e^{2.5t}\sin\sqrt{3.75}\,t & e^{2.5t}\cos\sqrt{3.75}\,t - \dfrac{0.5}{\sqrt{3.75}} e^{2.5t}\sin\sqrt{3.75}\,t \end{bmatrix} *$$

$$x_{(t)} = \Phi_{(t)} x_0 + \int_0^t \Phi_{(t-\tau)} \cancelto{0}{B u_{(\tau)}}\, d\tau \Rightarrow x_{(t)} = \Phi_{(t)} x_0 \Rightarrow$$

$$x_{(t)} = [\Phi_{(t)}]\begin{Bmatrix}1\\1\end{Bmatrix} = \begin{bmatrix} e^{2.5t}\cos\sqrt{3.75}\,t + \dfrac{1.5}{\sqrt{3.75}}\,e^{2.5t}\sin\sqrt{3.75}\,t \\ \\ e^{2.5t}\cos\sqrt{3.75}\,t - \dfrac{4.5}{\sqrt{3.75}}\,e^{2.5t}\sin\sqrt{3.75}\,t \end{bmatrix} \;**$$

$$y = [0 \quad 1]\begin{Bmatrix}x_1\\x_2\end{Bmatrix} = e^{2.5t}\left[\cos\sqrt{3.75}\,t - \frac{4.5}{\sqrt{3.75}}\sin\sqrt{3.75}\,t\right]$$