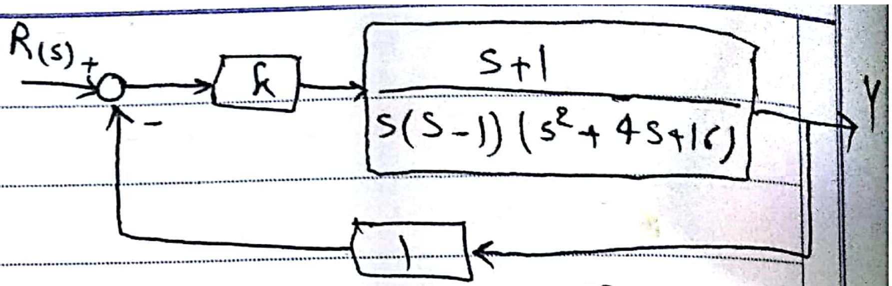
plot root lucos for



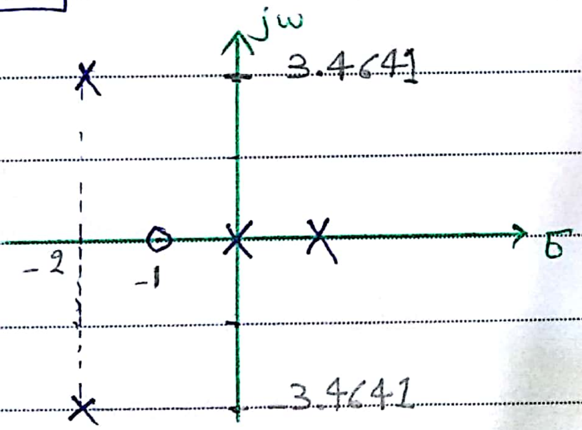
Assum that the value of gain k is nonnegative ($k \geq 0$)

$\boxed{1.1}$ calculate zeros and poles of open-loop transfer function.

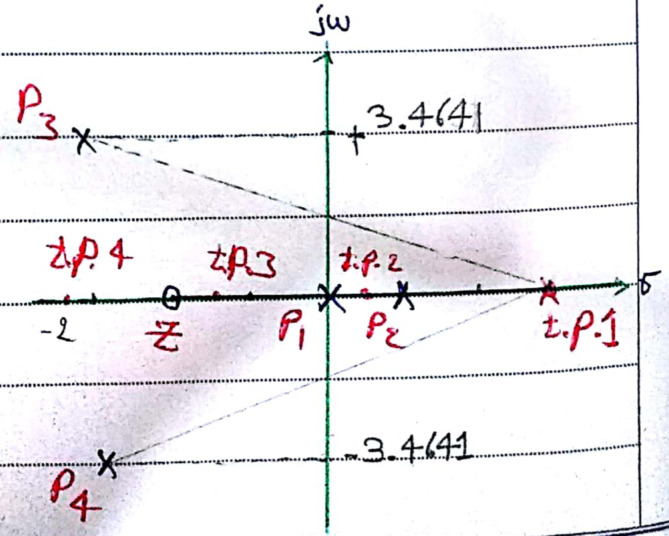
open-loop T.F. $= G_{(s)} H_{(s)} = \dfrac{s+1}{s(s+1)(s^2+4s+16)}$

$$\begin{cases} \text{zeros: } s=-1 \\ \text{poles: } s=0,\ s=1 \\ s=-2 \pm 3.4641i \end{cases}$$

$\boxed{1.2}$ locate the zeros and poles of open-loop T.F. on the s plane.



$\boxed{2}$: Determine the root loci on the real axis:



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محل مهر آموزش

t.p.1

$\angle P_1 = \angle P_2 = \angle Z = 0 \qquad \angle P_3 + \angle P_4 = 2\pi$

The angle condition is not satisfied for t.p.1

t.p.2 $\Rightarrow \angle P_2 = \pi,\ \angle P_1 = \angle Z = 0,\ \angle P_3 + \angle P_4 = 2\pi \Rightarrow 0 - 3\pi = \pm(2k+1)\pi$

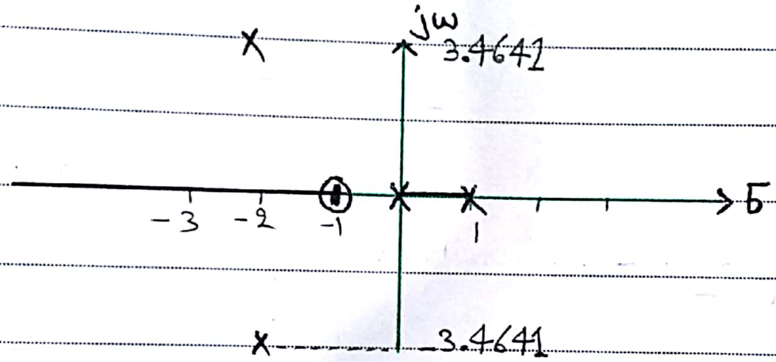
The angle condition is satisfied for t.p.2

t.p.3 : $\angle P_1 = \angle P_2 = \pi,\ \angle Z = 0,\ \angle P_3 + \angle P_4 = 2\pi \Rightarrow 0 - (\pi + 2\pi) \neq \pm(2k+1)\pi$

The angle condition is not satisfied for t.p.3

t.p.4 : $\angle P_1 = \angle P_2 = \angle Z = \pi,\ \angle P_3 + \angle P_4 = 2\pi \Rightarrow \pi - (\pi + \pi + 2\pi) = \pm(2k+1)\pi$

The angle condition is satisfied for t.p.4



$\boxed{3}$ Determin the asymptots of the root loci:

$\boxed{3.1}$ Determine number of asymptotes $\begin{cases} n = 4 \\ m = 1 \end{cases}$ $N = n - m = 3$

$\boxed{3.2}$ Determine the angle of asymptotes: $\phi_A = \dfrac{\pm\pi(2k+1)}{n-m}$

$$\phi_A = \frac{\pi(2k+1)}{4-1} = \frac{\pi}{3}(2k+1) \qquad k = 0, 1, 2$$

$\boxed{3.3}$ The asymptotes ~~inters~~ intersect at a point on the real axis:

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$$\sigma_A = \frac{(-2-2+1)-(-1)}{4-1} = -\frac{2}{3}$$

4 Find the breack away and break-in points

4.1 Write characteristic equation

$$1 + \frac{k(s+1) \alpha 1}{s(s-1)(s^2+4s+16)} = 0$$

4.2 obtain k form the characteristic equation

$$k = -\frac{s(s-1)(s^2+4s+16)}{(s+1)} = -\frac{s^4+3s^3+12s^2-16s}{(s+1)} \Rightarrow$$

4.3 Determines from $dk/ds = 0$

$$\frac{dk}{ds} = -\frac{(4s^3+9s^2+24s-16)(s+1) - (1)(s^4+3s^3+12s^2-16s)}{(s+1)^2} = 0$$

$$\Rightarrow \frac{dk}{ds} = \frac{3s^4+10s^3+21s^2+24s-16}{(s+1)^2} = 0 \Rightarrow$$

$s = 0.4483 \Rightarrow k = 3.0729$

$s = -0.7595 + 2.1637i$

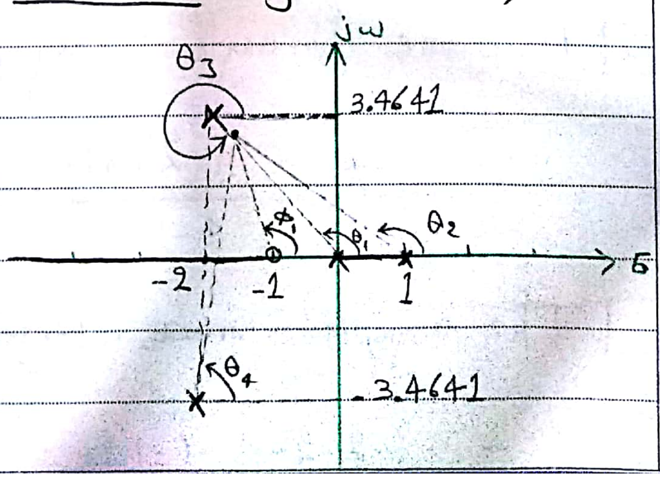
$s = -0.7595 - 2.1637i$

$\Rightarrow k = 30.2562 + 319.14$, -3.1914

$s = -2.2627 \Rightarrow k = 70.5628$

4.4 calculate k for s_1, s_2, s_3 and s_4

5 Determine the angle of departure (angle of arrival) of the root locus from frome a omplex poles (at a complex zeros



$\theta_1 = 180^\circ - tg^{-1}\left(\frac{3.4641}{2}\right) = 120^\circ$, $\theta_2 = 180^\circ - tg^{-1}\left(\frac{3.4641}{3}\right) = 130.89^\circ$

$\theta_3 = ?$, $\theta_4 \simeq 90^\circ$, $\Phi = \Phi_1 = 180 - tg^{-1}\left(\frac{3.4641}{1}\right) = 106.10^\circ$

$\Phi - (\theta_1 + \theta_2 + \theta_3 + \theta_4) = \pm(2k+1)180 \Rightarrow 106.10 - (120 + 130.89 + \theta_3 + 90) = \pm 180^\circ$

$$\Rightarrow \theta_3 = \begin{cases} -54.79 \longrightarrow \text{for } P_3 \\ 305.21 = +54.79^\circ \longrightarrow \text{for } P_4 \end{cases}$$

$\boxed{6}$ Find the points where the root locus cross the imaginary axis

$\boxed{6.1}$ Routh's stability criterion

3.19.14
3.1914

characteristic equation $\Rightarrow$ $s^4 + 3s^3 + 12s^2 + (k-16)s + k = 0$

s^4	1	12	k
s^3	3	$k-16$	0
s^2	$\frac{3\times 12-(k-16)}{3}$	k	0
s^1	$\dfrac{\frac{3\times 12-(k-16)}{3}\times(k-16)-3k}{\frac{3\times 12-(k-16)}{3}}$	0	0
$s0$	k	0	0

$k > 0 \Rightarrow -k^2 + 59k - 832 = 0 \Rightarrow \begin{cases} k_1 = 23.3153 \\ k_2 = 35.6847 \end{cases}$

Auxiliary equation

$$\frac{52-k}{3}s^2 + k \Rightarrow \begin{cases} 9.5615\,s^2 + 23.3133 = 0 \Rightarrow s = \pm i\,1.5616 \\ 5.4384\,s + 35.6847 = 0 \Rightarrow s = \pm 2.5617\,i \end{cases}$$

$\boxed{6.2}$ leting $s=j\omega$ in the characteristic equation

$(j\omega)^4+3(j\omega)^3+12(j\omega)^2+(k-16)(j\omega)+k=0$

$\omega^4-3j\omega^3-12\omega^2+(k-16)j\omega+k=0 \Rightarrow$

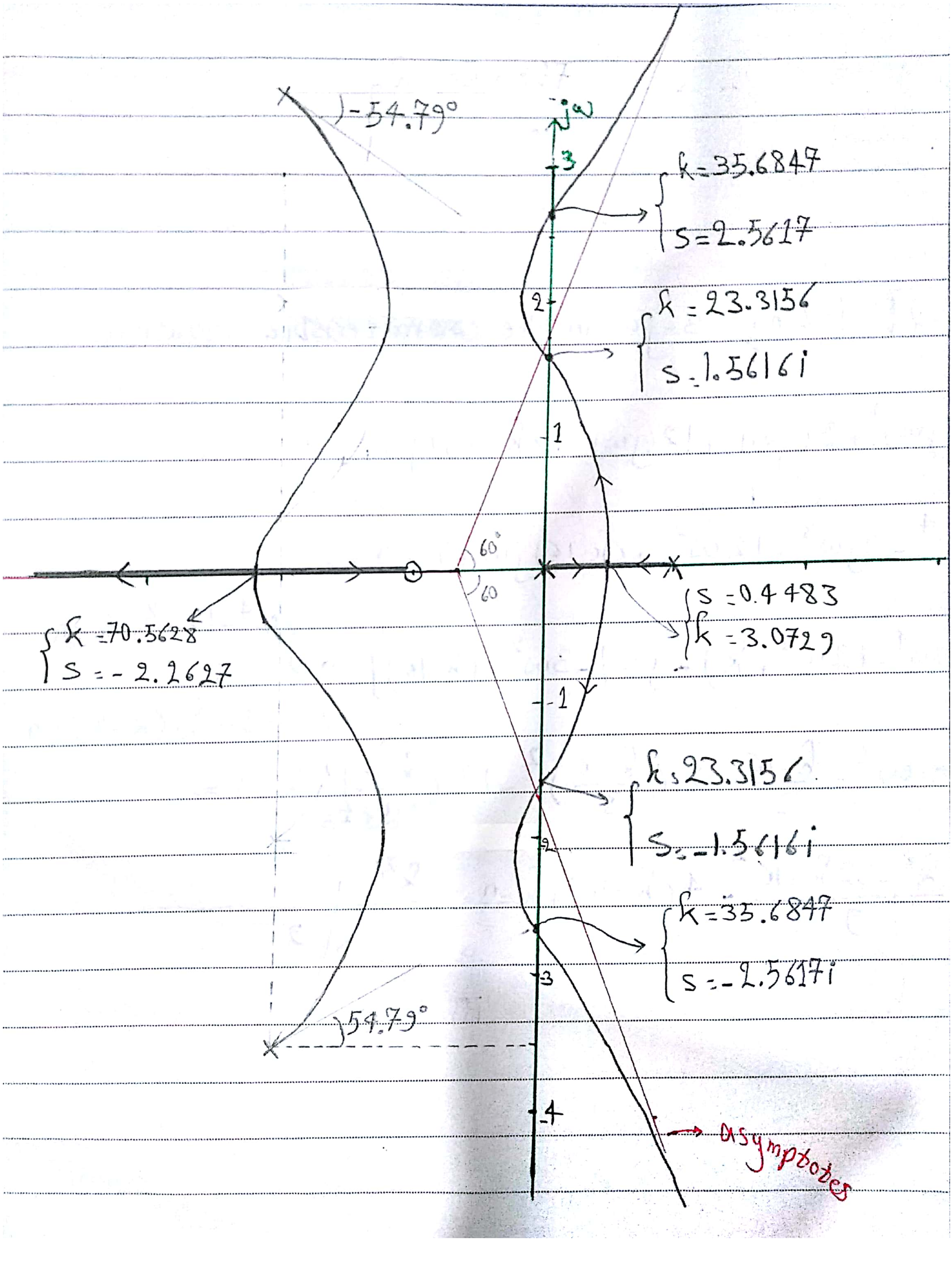
$(\omega^4-12\omega^2+k)+j\omega[-3\omega^2+(k-16)]=0 \begin{cases} \omega^4-12\omega^2+k=0 \\ -3\omega^2+(k-16)=0 \end{cases}$

$\Rightarrow \omega^2=\frac{k-16}{3} \Rightarrow \left(\frac{k-16}{3}\right)^2-12\left(\frac{k-16}{3}\right)+k=0 \Rightarrow$

$\frac{k^2-32k+16^2}{9}-4(k-16)+k=0 \Rightarrow \frac{k^2-59k+832}{9}=0 \Rightarrow$

$k_1=23.3153 \Rightarrow -3\omega^2+(23.3153-16)=0 \Rightarrow \omega=\pm 1.5616i$

$k_2=35.6847 \Rightarrow -3\omega^2+(35.6847-16)=0 \Rightarrow \omega=2.5617i$



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