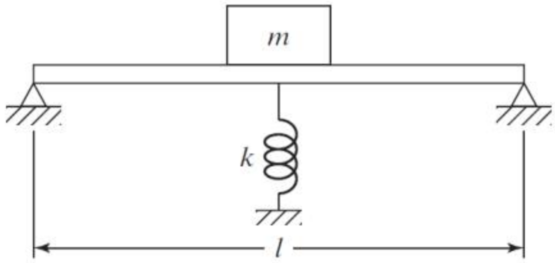
Vibration quiz

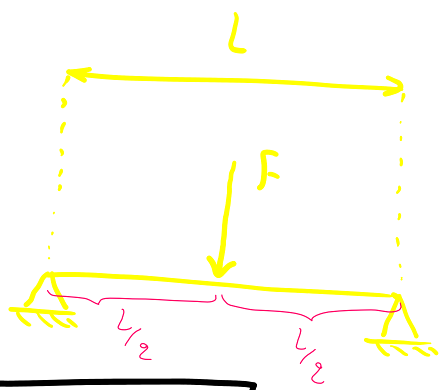
Tuesday, February 15, 2022 11:15 PM

1.11 A machine of mass $m = 500$ kg is mounted on a simply supported steel beam of length $l = 2$ m having a rectangular cross section (depth $= 0.1$ m, width $= 1.2$ m) and Young's modulus $E = 2.06 \times 10^{11}$ N/m^2. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at mid-span, as shown in Fig. 1.71. Determine the value of k needed to reduce the deflection of the beam by

a. 25 percent of its original value.
b. 50 percent of its original value.
c. 75 percent of its original value.

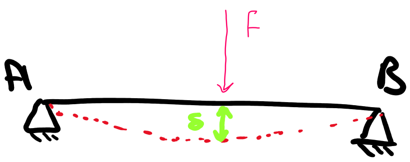
Assume that the mass of the beam is negligible.

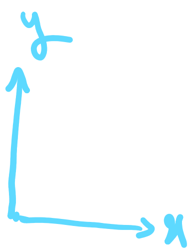




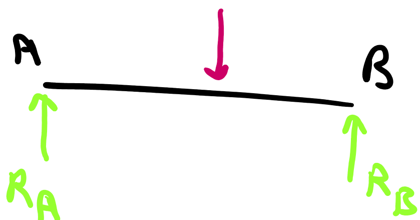
$$\boxed{I = \frac{1}{12} w d^4}$$

$E =$





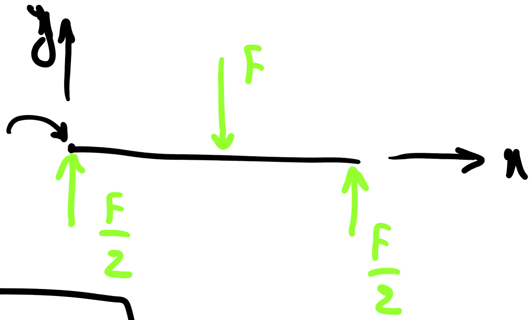
F.B.D.



$$\Sigma F_y = 0 \Rightarrow R_A + R_B - F = 0 \Rightarrow \underline{R_A + R_B = F} \quad (1)$$

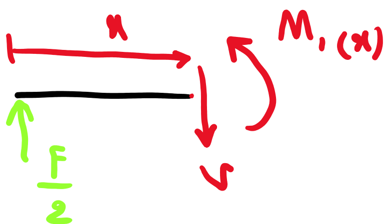
$$\sum M)_B = 0 \Rightarrow F \times \frac{l}{2} - R_A l = 0 \Rightarrow \boxed{R_A = \frac{1}{2}F} \quad (2)$$

(2) into (1) $\Rightarrow \frac{1}{2}F + R_B = F \Rightarrow \boxed{R_B = \frac{1}{2}F}$ (3)



$$\boxed{EI\frac{d^2y}{dx^2} = M(x)} \quad (4)$$

$0 \le x < \frac{l}{2}$



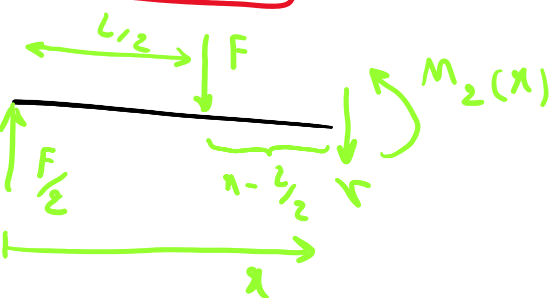
$$M_1(x) - \frac{F}{2}x = 0 \Rightarrow M_1(x) = \frac{1}{2}Fx \quad (5)$$

(5) into (4) $\Rightarrow EI\frac{d^2y}{dx^2} = \frac{1}{2}Fx \Rightarrow$

$$EI\frac{dy}{dx} = EI\theta = \frac{1}{4}Fx^2 + C_1 \quad (6)$$

$$EIy = \frac{1}{12}Fx^3 + C_1x + C_2 \quad (7)$$

$\frac{l}{2} \le x \le l$



$$M_2(x) + F\left(x - \frac{l}{2}\right) - \frac{F}{2} \times x = 0 \Rightarrow$$

$$M_2(x) + F\left(x - \frac{L}{2}\right) - \frac{F}{2} \times x = 0 \Rightarrow$$

$$\boxed{M_2(x) = -\frac{1}{2}Fx + \frac{1}{2}FL} \quad (8)$$

(8) into (4) $\Rightarrow EI\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}Fx + \dfrac{1}{2}FL \Rightarrow$

$$\boxed{EI\theta = EI\frac{dy}{dx} = -\frac{1}{4}Fx^2 + \frac{1}{2}FLx + C_3} \quad (9) \Rightarrow$$

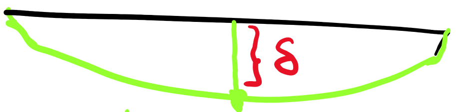
$$\boxed{EIy = -\frac{1}{12}Fx^3 + \frac{1}{4}FLx^2 + C_3x + C_4} \quad (10)$$

we can calculate unknowns C_1, C_2, C_3, and C_4 using boundary conditions.

$$\begin{cases} \text{B.C.1: } y\big|_{x=0} = 0 \\ \text{B.C.2: } y\big|_{x=L} = 0 \end{cases}$$

B.C.3: $y\big|_{L/2^-} = y\big|_{L/2^+}$

B.C.4: $\theta\big|_{L/2^-} = \theta\big|_{L/2^+}$



$y\big|_{L/2^-} = y\big|_{L/2^+}$

(7)

$EIy = \frac{1}{12}Fx^3 + C_1x + C_2$ → $y = \frac{1}{EI}\left[\frac{1}{12}Fx^3 + C_1x + C_2\right]$ and B.C.1

$$x = 0 \Rightarrow y = \frac{1}{EI}\left[\frac{1}{12}F \times 0^3 + C_1 \times 0 + C_2\right] = 0 \Rightarrow$$

$$\boxed{C_2 = 0} \quad (11)$$

$$\boxed{EIy = -\frac{1}{12}Fx^3 + \frac{1}{4}Flx^2 + C_3 x + C_4} \quad (10)$$

B.C.2 $\Rightarrow y(x=l)=0 \Rightarrow$

$$y = \frac{1}{EI}\left[-\frac{1}{12}Fx^3 + \frac{1}{4}Flx^2 + C_3 x + C_4\right] \Rightarrow$$

$$0 = \frac{1}{EI}\left[-\frac{1}{12}Fl^3 + \frac{1}{4}Fl^3 + C_3 l + C_4\right] \Rightarrow$$

$$\boxed{C_3 l + C_4 = -\frac{1}{6}Fl^3} \quad (12)$$

(7), (10) and B.C.3

$$\underbrace{\frac{1}{EI}}\left[\frac{1}{12}F\left(\frac{l}{2}\right)^3 + C_1\left(\frac{l}{2}\right)\right] = \underbrace{\frac{1}{EI}}\left[-\frac{1}{12}F\left(\frac{l}{2}\right)^3 + \frac{1}{16}Fl^3 + C_3\frac{l}{2} + C_4\right]$$

$$\boxed{-\frac{1}{2}C_1 l + \frac{1}{2}C_3 l + C_4 = -\frac{1}{24}Fl^3} \quad (13)$$

Eq.s (6), (9) and B.C. 4

$$\frac{1}{EI}\left[\frac{1}{4}F\left(\frac{l}{2}\right)^2 + C_1\right] = \frac{1}{EI}\left[-\frac{1}{4}F\left(\frac{l}{2}\right)^2 + \frac{1}{4}Fl^2 + C_3\right]$$

$$\Rightarrow \boxed{C_1 - C_3 = \frac{1}{8}Fl^2} \quad (14)$$

$$\begin{cases} c_1 \\ c_2 \\ c_3 \\ c_4 \end{cases} \quad \text{and} \quad \begin{cases} (11) \longrightarrow c_1 = 0 \\ (12) \\ (13) \\ (14) \end{cases}$$

$$\begin{cases} c_3 l + c_4 = -\frac{1}{6} F l^3 \implies c_4 = -\frac{1}{6} F l^3 - c_3 l \\ -\frac{1}{2} c_1 l + \frac{1}{2} c_3 l + c_4 = -\frac{1}{24} F l^3 \\ c_1 - c_3 = \frac{1}{8} F l^2 \implies c_1 = c_3 + \frac{1}{8} F l^2 \end{cases}$$

$$-\frac{1}{2}\left[c_3 + \frac{1}{8} F l^2\right] l + \frac{1}{2} c_3 l + \left[-\frac{1}{6} F l^3 - c_3 l\right] = -\frac{1}{24} F l$$

$$-\frac{1}{16} F l^3 - \frac{1}{6} F l^3 - c_3 l = -\frac{1}{24} F l^3 \implies$$

$$\boxed{c_3 = -\frac{3}{16} F l^2} \quad (15)$$

(15) into (14) $\implies c_1 - \left[-\frac{3}{16} F l^2\right] = \frac{1}{8} F l^2 \implies$

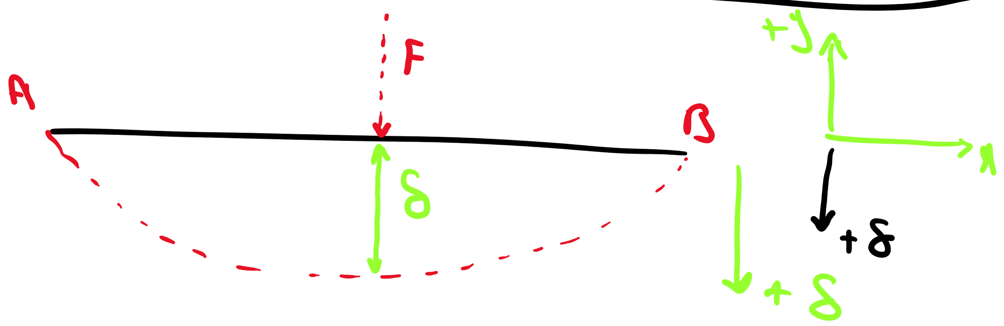
$$\boxed{c_1 = -\frac{1}{16} F l^2} \quad (16)$$

$$c_4 = -\frac{1}{6} F l^3 - \left[-\frac{3}{16} F l^2\right] l \implies$$

$$\boxed{C_4 = \frac{1}{48} F l^3} \quad (17)$$

(7), (11), (16) $\Rightarrow$

$$\boxed{y = \frac{1}{EI}\left[\frac{1}{12} F x^3 - \frac{1}{16} F l^2 x\right] \quad 0 \leq x < l/2}$$



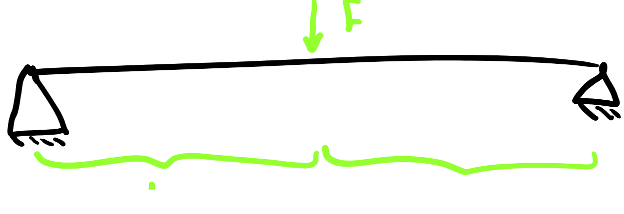
$$y\Big|_{x=\frac{l}{2}} = y_{(\frac{l}{2})} = \frac{1}{EI}\left[\frac{1}{12} F \left(\frac{l}{2}\right)^3 - \frac{1}{16} F l^2 \left(\frac{l}{2}\right)\right]$$

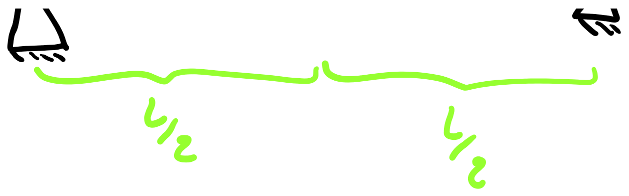
$$y = \frac{1}{EI}\left[\frac{1}{96} F l^3 - \frac{1}{32} F l^3\right] = -\frac{1}{48} F l^3$$

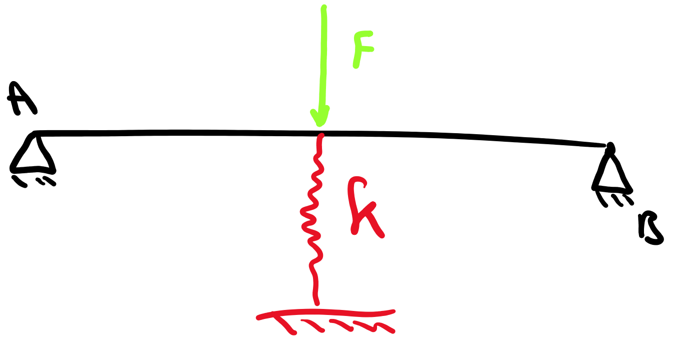
$$\delta = -y_{(\frac{l}{2})} = \frac{1}{48}\frac{F l^3}{EI} \Rightarrow k = \frac{F}{\delta} = \frac{48EI}{l^3}$$

$$\boxed{k_1 = \frac{48EI}{l^3}}$$

equivalant spring of this beam under F



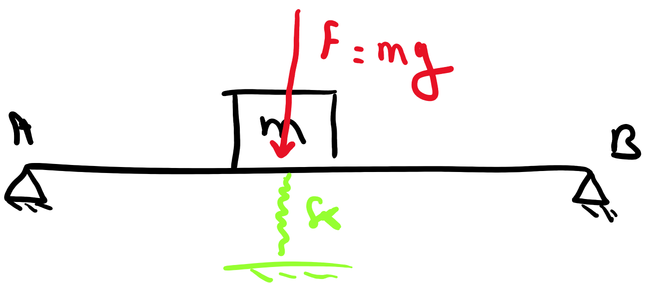




$$\boxed{k_{eq} = k + k_1}$$

Determine the value of $\underline{k}$ needed to reduce the deflection of the beam by 25 percent of its original value.

original value $= \delta = \frac{1}{48EI} F L^3$



original value: $\delta = \frac{1}{48EI} (mg) L^3$

$\delta_1 = \frac{25}{100} \delta$ or $\boxed{\delta_1 = G\delta}$ $G = 0.25, 0.50, 0.75$

$$\delta_1 = \frac{1 \times G}{48EI} (mg) L^3$$

$$(k_1+k)\delta_1 = mg \Rightarrow k = \frac{mg}{\delta_1} - k_1 \Rightarrow$$

$$k = \frac{mg}{\frac{G}{48EI} mg\, l^3} - \frac{48EI}{l^3} = \frac{48EI}{G l^3} - \frac{48EI}{L^3}$$

$$\boxed{k = \frac{48EI}{l^3}\left(\frac{1}{G} - 1\right)} \quad \text{or}$$

$$\boxed{k = \frac{48EI}{l^3}\left(\frac{1-G}{G}\right)}, \; G = 0.25, 0.50, 0.75$$

$\boxed{k \text{ is independent of } m}$ Note

$d = 0.1\ m$

$w = 1.2\ m$

$L = 2\ m$

$E = 2.06 \times 10^{11}\ \frac{N}{m^2}$

$m = 500\ kg$

$$I = \frac{1}{12} w d^3 = \frac{1}{12} \times 1.2 \times 0.1^3 = 1 \times 10^{-4}\ m^4$$

$\delta_1 = 0.25\,\delta$ or $G = 0.25$

$$k = \frac{48 \times 2.06 \times 10^{11} \times 10^{-4}}{2^3}\left(\frac{1-0.25}{0.5}\right) = 3.708 \times 10^8\ \frac{N}{m}$$

$\delta_1 = 0.50\,\delta$ or $G = 0.50$

$$k = \frac{48 \times 2.06 \times 10^{11} \times 10^{-4}}{2^3}\left(\frac{1-0.50}{}\right) = 1.236 \times 10^8\ \frac{N}{m}$$

$$k = \frac{48 \times 2.06 \times 10^{11} \times 10^{-4}}{2^3} \left(\frac{1 - 0.50}{0.5} \right) = 1.236 \times 10^8 \ N/m$$

$$\delta_1 = 0.75\delta \quad \text{or} \quad G = 0.75$$

$$k = \frac{48 \times 2.06 \times 10^{11} \times 10^{-4}}{2^3} \left(\frac{1 - 0.75}{0.75} \right) = 4.12 \times 10^7 \ N/m$$
$$= 0.412 \times 10^8 \ N/m$$