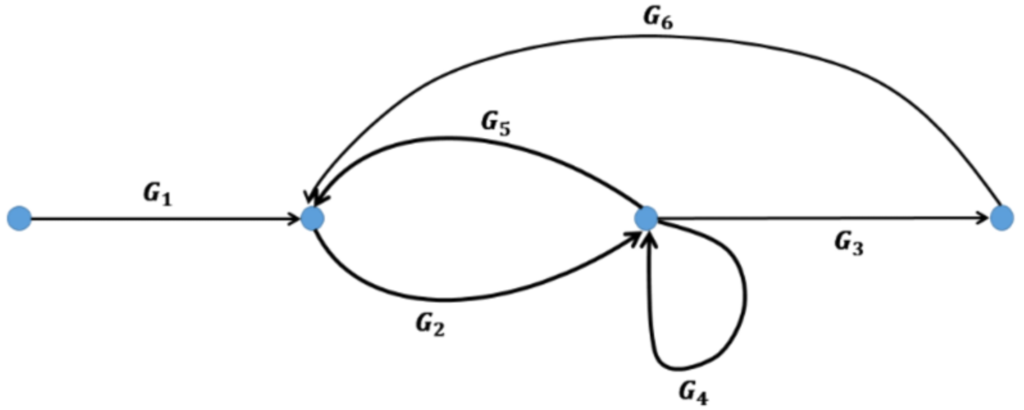
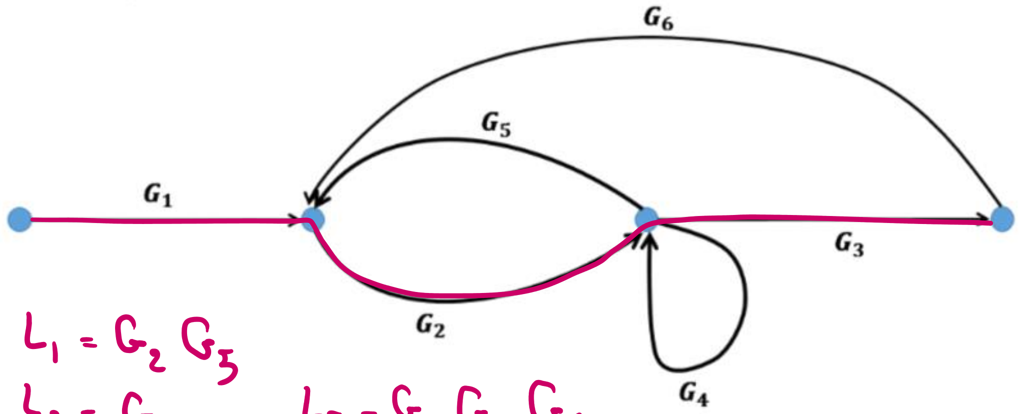
Control midterm q2

Thursday, December 2, 2021 5:51 PM

Using **Mason's signal-flow gain formula**, obtain the transfer function.





$L_1 = G_2 G_3$

$L_2 = G_4$ $\quad L_3 = G_2 G_3 G_6$

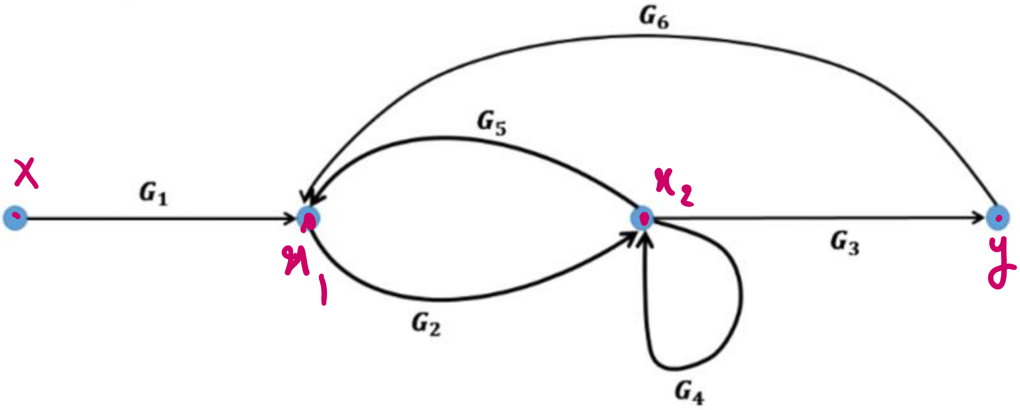
$P_1 = G_1 G_2 G_3$ $\quad \Delta_1 = 1$

$\Delta = 1 - (G_2 G_3 + G_4 + G_2 G_3 G_6)$ $\Rightarrow$

$$T = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3}{1 - (G_2 G_3 + G_4 + G_2 G_3 G_6)}$$

$$\boxed{T = \frac{G_1 G_2 G_3}{1-(G_2G_3 + G_4 + G_2G_3G_6}}$$

Naming Method



$$\begin{cases} x_1 = G_1 X + G_5 x_2 + G_6 Y & (1) \\ x_2 = G_2 x_1 + G_4 x_2 \Rightarrow x_2 = \dfrac{G_2}{1-G_4} x_1 & (2) \\ Y = G_3 x_2 & (3) \end{cases}$$

$$(2),(1) \Rightarrow x_2 = \frac{G_2}{1-G_4}\left[G_1 X + G_5 x_2 + G_6 Y\right] \Rightarrow$$

$$\left(1 - \frac{G_2 G_5}{1-G_4}\right) x_2 = \frac{G_1 G_2}{1-G_4} X + \frac{G_2 G_6}{1-G_4} Y \Rightarrow$$

$$(1 - G_4 - G_2 G_5) x_2 = G_1 G_2 X + G_2 G_6 Y \Rightarrow$$

$$x_2 = \frac{G_1 G_2}{1-G_4-G_2G_5} X + \frac{G_2 G_6}{1-G_4-G_2G_5} Y \qquad (4)$$

$$(3),(4) \Rightarrow Y = G_3 x_2$$

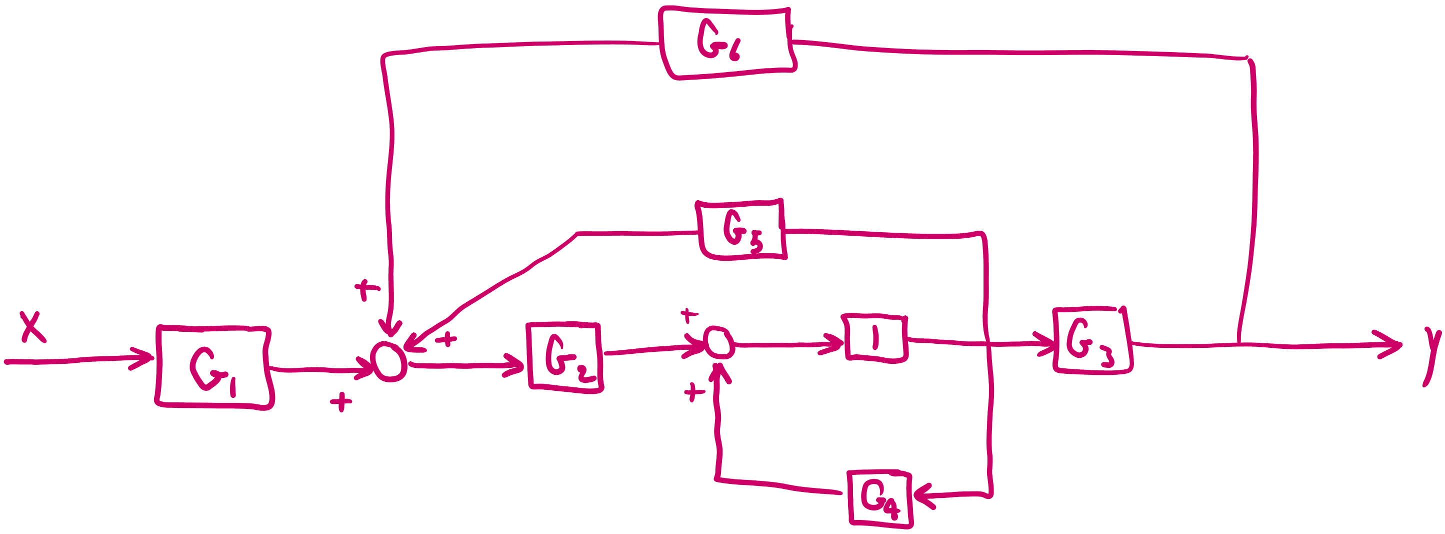
$$y = \frac{G_1 G_2 G_3}{1 - G_4 - G_2 G_5} X + \frac{G_2 G_3 G_6}{1 - G_4 - G_2 G_5} y \Longrightarrow$$

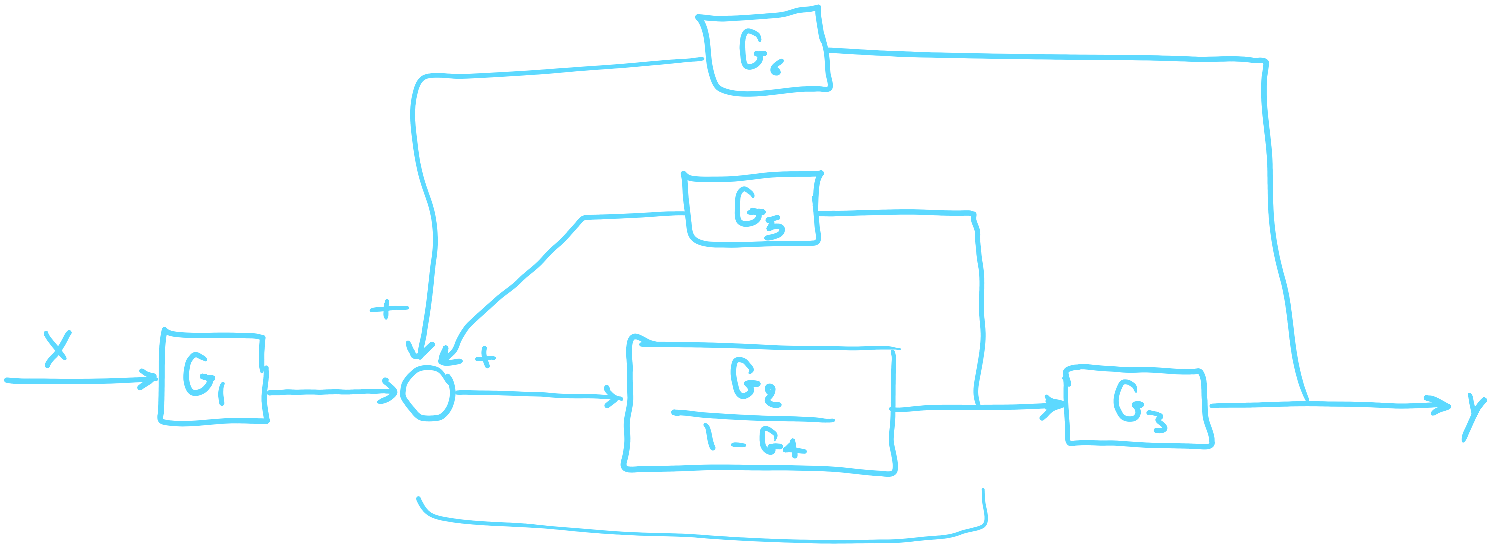
$$\left(1 - \frac{G_2 G_3 G_6}{1 - G_4 - G_2 G_5}\right) y = \frac{G_1 G_2 G_3}{1 - G_4 - G_2 G_5} X \Rightarrow$$

$$(1 - G_4 - G_2 G_5 - G_2 G_3 G_6) y = G_1 G_2 G_3 X \Rightarrow$$

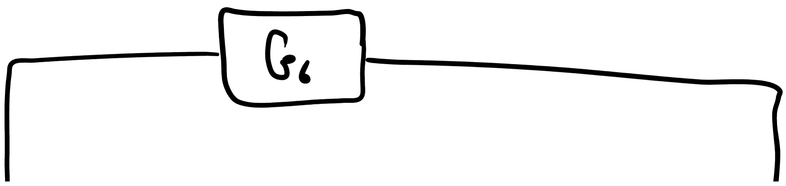
$$\boxed{T = \frac{y}{X} = \frac{G_1 G_2 G_3}{1 - [G_4 + G_2 G_5 + G_2 G_3 G_6]}}$$

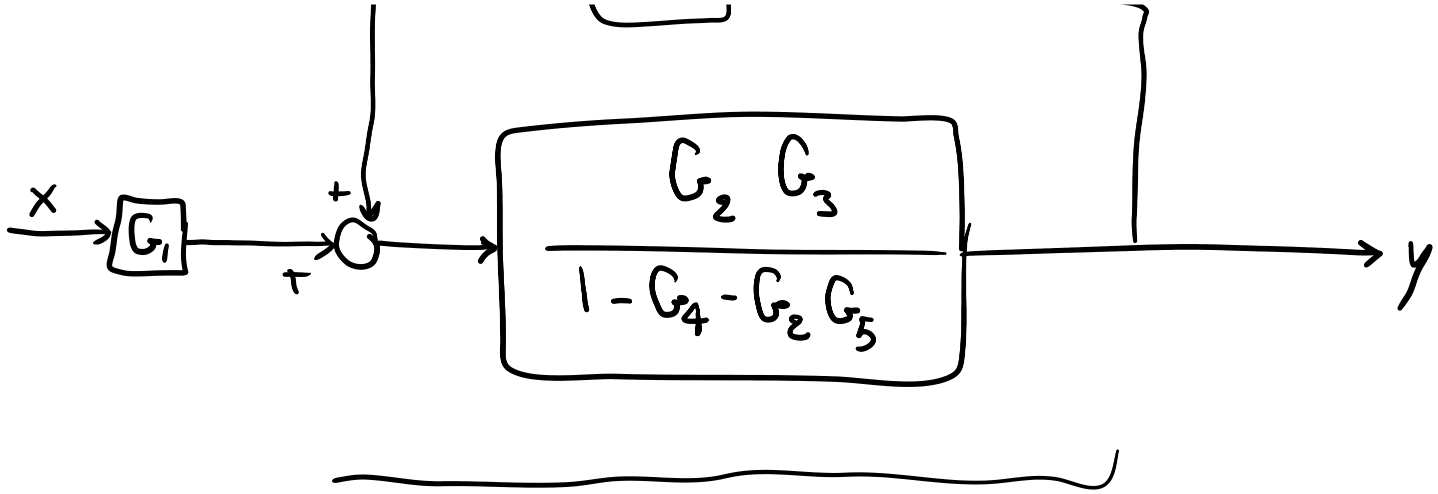
Equivalent Block Diagram



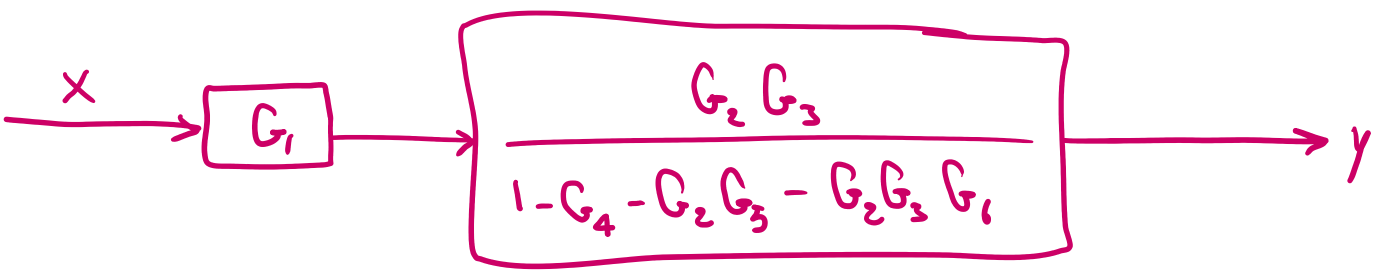


$$\frac{\frac{G_2}{1-G_4}}{1-\frac{G_5 G_2}{1-G_4}} = \frac{G_2}{1-G_4-G_2 G_5}$$





$$\frac{\dfrac{G_2G_3}{1-G_4-G_2G_5}}{1-\dfrac{G_2G_3G_6}{1-G_4-G_2G_5}} = \frac{G_2G_3}{1-G_4-G_2G_5-G_2G_3G_6}$$



$$\boxed{T = \frac{Y}{X} = \frac{G_1 G_2 G_3}{1 - G_4 - G_2 G_3 - G_2 G_3 G_6}}$$