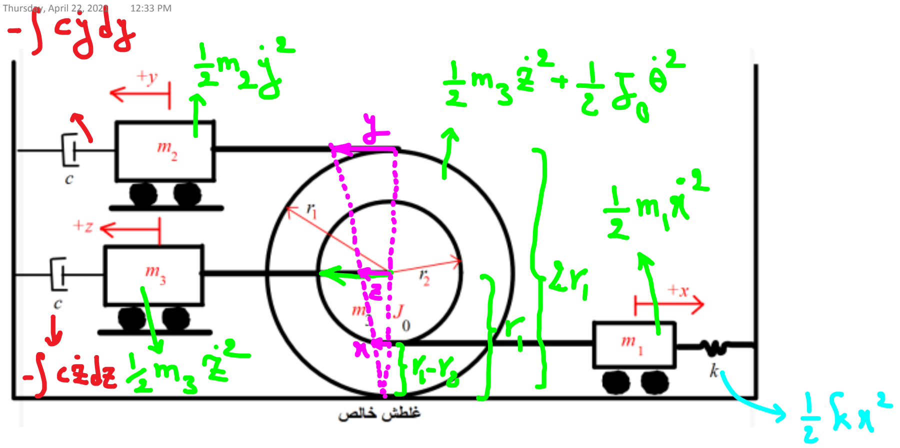
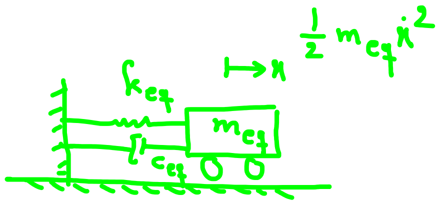




$y = 2r_1\theta$

با استفاده از انرژی جنبشی جرم معادل را محاسبه می کنیم:

$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\dot{y}^2+\frac{1}{2}m_3\dot{z}^2+\left[\frac{1}{2}m_3\dot{z}^2+\frac{1}{2}J_0\dot{\theta}^2\right]$$

رابطه بین $\dot{y}$ ، $\dot{z}$ و $\dot{\theta}$ برحسب $\dot{x}$ پیدا کنیم

$$\frac{x}{r_1-r_2}=\frac{z}{r_1}=\frac{y}{2r_1} \Rightarrow \begin{cases} y=\dfrac{2r_1}{r_1-r_2}x \\ z=\dfrac{r_1}{r_1-r_2}x \end{cases}$$

$$x=(r_1-r_2)\theta \Rightarrow \theta=\frac{x}{r_1-r_2} \Rightarrow$$

$$\frac{1}{2}m_{eq}\dot{x}^2=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\left(\frac{2r_1}{r_1-r_2}\dot{x}\right)^2+\frac{1}{2}m_3\left(\frac{r_1}{r_1-r_2}\dot{x}\right)^2+\left[\frac{1}{2}m_3\left(\frac{r_1}{r_1-r_2}\dot{x}\right)^2+\frac{1}{2}J_0\left(\frac{\dot{x}}{r_1-r_2}\right)^2\right]$$

$$x:\quad m_{eq}=m_1+\left(\frac{2r_1}{r_1-r_2}\right)^2m_2+\left(\frac{r_1}{r_1-r_2}\right)^2m_3+\left(\frac{r_1}{r_1-r_2}\right)^2m_3+\left(\frac{1}{r_1-r_2}\right)^2J_0 \Rightarrow$$

$$\boxed{m_{eq}=m_1+\left(\frac{1}{r_1-r_2}\right)^2\left[r_1^2(4m_2+2m_3)+J_0\right]}$$

جرم معادل در راستای x

$$\frac{1}{2}m_{eq}\dot{y}^2=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\dot{y}^2+\frac{1}{2}m_3\dot{z}^2+\left[\frac{1}{2}m_3\dot{z}^2+\frac{1}{2}J_0\dot{\theta}^2\right]$$

$$\frac{x}{r_1-r_2}=\frac{z}{r_1}=\frac{y}{2r_1} \Rightarrow \begin{cases} x=\dfrac{r_1-r_2}{2r_1}y \\ z=\dfrac{r_1}{2r_1}y=\dfrac{1}{2}y \end{cases}$$

$$y=2r_1\theta \Rightarrow \theta=\frac{y}{2r_1}$$

$$\frac{1}{2}m_{eq}\dot{y}^2=\frac{1}{2}m_1\left(\frac{r_1-r_2}{2r_1}\dot{y}\right)^2+\frac{1}{2}m_2\dot{y}^2+\frac{1}{2}m_3\left(\frac{1}{2}\dot{y}\right)^2+\left[\frac{1}{2}m_3\left(\frac{1}{2}\dot{y}\right)^2+\frac{1}{2}J_0\left(\frac{\dot{y}}{2r_1}\right)^2\right] \Rightarrow$$

$$\hookrightarrow \frac{1}{2} m_{eq}\dot{y}^2 = \frac{1}{2} m_1\left(\frac{r_1-r_2}{2r_1}\dot{y}\right)^2 + \frac{1}{2} m_2 \dot{y}^2 + \frac{1}{2} m_3\left(\frac{1}{2}\dot{y}\right)^2 + \left[\frac{1}{2} m_3\left(\frac{1}{2}\dot{y}\right)^2 + \frac{1}{2} J_0\left(\frac{\dot{y}}{2r_1}\right)^2\right] \Rightarrow$$

$$m_{eq} = \left(\frac{r_1-r_2}{2r_1}\right)^2 m_1 + m_2 + \frac{1}{4} m_3 + \frac{1}{4} m_3 + \frac{1}{4r_1^2} J_0 \Rightarrow$$

$$\boxed{y: \; m_{eq} = \left(\frac{r_1-r_2}{2r_1}\right)^2 m_1 + m_2 + \frac{1}{2} m_3 + \frac{1}{4}\frac{J_0}{r_1^2}}$$

جرم معادل در راستای y

محاسبه سختی معادل با استفاده از انرژی پتانسیل صورت می‌گیرد.

$$\frac{1}{2} k_{eq} x^2 = \frac{1}{2} k x^2 \Rightarrow \boxed{x: k_{eq} = k}$$

سختی معادل در راستای x

$$y: \frac{1}{2} k_{eq} y^2 = \frac{1}{2} k x^2 \Rightarrow \frac{1}{2} k_{eq} y^2 = \frac{1}{2} k \left(\frac{r_1-r_2}{2r_1} y\right)^2 \Rightarrow \boxed{k_{eq} = \left(\frac{r_1-r_2}{2r_1}\right)^2 k}$$

سختی معادل در راستای y

- محاسبه میرایی معادل با استفاده از انرژی تلف شده انجام می‌شود.

$$x: -\int c_{eq}\dot{x}\,dx = -\int c\dot{y}\,dy - \int c\dot{z}\,dz$$

$$\frac{x}{r_1-r_2} = \underbrace{\frac{z}{r_1} = \frac{y}{2r_1}} \Rightarrow \begin{cases} y = \frac{2r_1}{r_1-r_2} x \\ z = \frac{r_1}{r_1-r_2} x \end{cases} \Rightarrow \begin{cases} \dot{y} = \frac{2r_1}{r_1-r_2}\dot{x}, & dy = \frac{2r_1}{r_1-r_2} dx \\ \dot{z} = \frac{r_1}{r_1-r_2}\dot{x}, & dz = \frac{r_1}{r_1-r_2} dx \end{cases}$$

$$-\int c_{eq}\dot{x}\,dx = -\int c\left(\frac{2r_1}{r_1-r_2}\dot{x}\right)\left(\frac{2r_1}{r_1-r_2}dy\right) - \int c\left(\frac{r_1}{r_1-r_2}\dot{x}\right)\left(\frac{r_1}{r_1-r_2}dx\right) \Rightarrow$$

$$x: \; c_{eq} = \left(\frac{2r_1}{r_1-r_2}\right)^2 c + \left(\frac{r_1}{r_1-r_2}\right)^2 c \Rightarrow \boxed{c_{eq} = 5\left(\frac{r_1}{r_1-r_2}\right)^2 c}$$

دمپر معادل در راستای x

$$y: -\int c_{eq}\dot{y}\,dy = -\int c\dot{y}\,dy - \int c\dot{z}\,dz \quad \text{and} \quad z = \frac{1}{2}y \Rightarrow \dot{z} = \frac{1}{2}\dot{y} \quad \text{and} \quad dz = \frac{1}{2}dy$$

$$\Rightarrow -\int c_{eq}\dot{y}\,dy = -\int c\dot{y}\,dy - \int c\left(\frac{1}{2}\dot{y}\right)\left(\frac{1}{2}dy\right) \Rightarrow c_{eq} = c + \frac{1}{4}c \Rightarrow \boxed{c_{eq} = \frac{5}{4}c}$$

ادامه معادلات درس

- محاسبه فرکانس طبیعی

$$x: \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} \Rightarrow$$

$$\omega_n\Big)_x = \sqrt{\frac{k}{m_1 + \left(\frac{1}{r_1 - r_2}\right)^2 \left[r_1^2 (4m_2 + 2m_3) + J_0 \right]}}$$

فرکانس طبیعی برای سیستم معادل سازی شده در x

$$y: \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}}$$

$$\omega_n\Big|_y = \sqrt{\frac{\left(\frac{r_1 - r_2}{2r_1}\right)^2 k}{\left(\frac{r_1 - r_2}{2r_1}\right)^2 m_1 + m_2 + \frac{1}{2} m_3 + \frac{1}{4} \frac{J_0}{r_1^2}}}$$

$$\omega_n\Big|_y = \left[\frac{\frac{\left(\frac{r_1 - r_2}{2r_1}\right)^2}{\left(\frac{r_1 - r_2}{2r_1}\right)^2} k}{\frac{\left(\frac{r_1 - r_2}{2r_1}\right)^2}{\left(\frac{r_1 - r_2}{2r_1}\right)^2} m_1 + \frac{1}{\left(\frac{r_1 - r_2}{2r_1}\right)^2} m_2 + \frac{1}{2} \frac{1}{\left(\frac{r_1 - r_2}{2r_1}\right)^2} m_3 + \frac{1}{4} \frac{1}{\left(\frac{r_1 - r_2}{2r_1}\right)^2} \frac{J_0}{r_1^2}} \right]^{1/2} \Rightarrow$$

$$\omega_n\Big|_y = \left[\frac{k}{m_1 + \left(\frac{2r_1}{r_1 - r_2}\right)^2 m_2 + \frac{1}{2}\left(\frac{2r_1}{r_1 - r_2}\right)^2 m_3 + \frac{1}{4}\left(\frac{2r_1}{r_1 - r_2}\right)^2 \frac{J_0}{r_1^2}} \right]^{1/2} \Rightarrow$$

$$\omega_n\Big|_y = \left[\frac{k}{m_1 + \frac{4r_1^2}{(r_1-r_2)^2}m_2 + \frac{1}{2}\frac{4r_1^2}{(r_1-r_2)^2}m_3 + \frac{1}{4}\frac{4r_1^2}{(r_1-r_2)^2}\frac{J_0}{r_1^2}}\right]^{\frac{1}{2}} \Rightarrow$$

$$\omega_n\Big|_y = \left[\frac{k}{m_1 + \left(\frac{1}{r_1-r_2}\right)^2\left[r_1^2(4m_2+2m_3)+J_0\right]}\right]^{\frac{1}{2}}$$

$$\omega_n\Big)_x = \sqrt{\frac{k}{m_1 + \left(\frac{1}{r_1-r_2}\right)^2\left[r_1^2(4m_2+2m_3)+J_0\right]}}$$

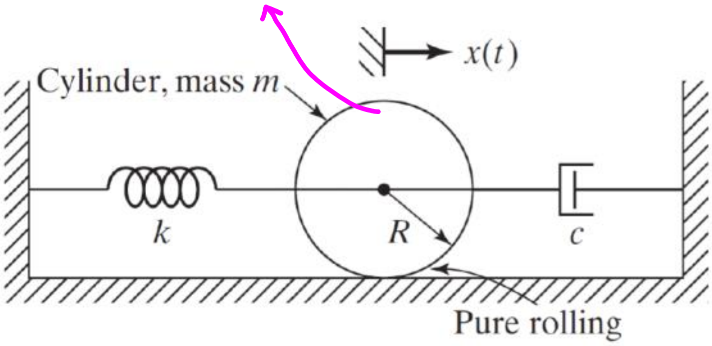
مقایسه $\omega_n|_x$ و $\omega_n|_y$ نشان می‌دهد که این دو فرکانس طبیعی باهم برابر هستند.

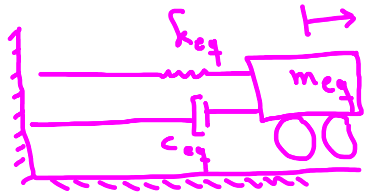
H.W.: جرم، فنر و دمپر معادل را در راستای z محاسبه کنید.

فرکانس طبیعی برای سیستم معادل سازی شده در راستای z را بدست آورید.

$\omega_n|_z$ را با $\omega_n|_x$ و $\omega_n|_y$ مقایسه کنید. دلیل تفاوت را توضیح دهید.

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 \qquad x = R\theta \Rightarrow \theta = \frac{x}{R} \Rightarrow \dot{\theta} = \frac{\dot{x}}{R}$$



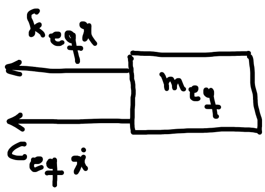


$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 \Rightarrow \frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\left(\frac{\dot{x}}{R}\right)^2 \Rightarrow m_{eq} = m + \frac{J}{R^2}$$

$$\text{and } J = \frac{1}{2}mR^2 \Rightarrow m_{eq} = m + \frac{\frac{1}{2}mR^2}{R^2} = \frac{3}{2}m \Rightarrow \boxed{m_{eq} = \frac{3}{2}m} \Rightarrow m_{eq} = \frac{3}{2}10 = 15\ \text{kg}$$

$$\frac{1}{2}k_{eq}x^2 = \frac{1}{2}kx^2 \Rightarrow \boxed{k_{eq} = k} \Rightarrow k_{eq} = k = 2000\ N/m$$

$$-\int c_{eq}\dot{x}\,dx = -\int c\dot{x}\,dx \Rightarrow \boxed{c_{eq} = c} \Rightarrow c_{eq} = c = 10\ \frac{N.s}{m}$$



$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = 0$$

$$\frac{3}{2}m\ddot{x} + c\dot{x} + kx = 0$$

معادله حرکت $\boxed{15\ddot{x} + 10\dot{x} + 2000x = 0}$

فرض می‌کنیم

$$x = ae^{st}$$

$$\Rightarrow \dot{x} = ase^{st} \Rightarrow \ddot{x} = as^2e^{st} \Rightarrow$$

$$15(as^2e^{st}) + 10(ase^{st}) + 2000(ae^{st}) = 0 \Rightarrow 15s^2 + 10s + 2000 = 0 \Rightarrow$$

$$s_1, s_2 = \frac{-10 \pm \sqrt{10^2 - 4\times15\times2000}}{2\times15} = \frac{-100 \pm \sqrt{-119000}}{30} = -3.333 \pm i\,11.5421\,9313$$

$$\boxed{s_1, s_2 \simeq -3.333 \pm i\,11.542}$$

$$x_{(t)} \equiv x = a_1e^{s_1t} + a_2e^{s_2t} = a_1e^{(-3.33+11.542i)t} + a_2e^{(-3.333-11.542i)t}$$

$$= e^{-3.333t}\left[a_1e^{11.542it} + a_2e^{-11.542it}\right]$$

$$= e^{-3.333t}\left[a_1\left(\cos(11.542t) + i\sin(11.542t)\right) + a_2\left(\cos(11.542t) - i\sin(11.542t)\right)\right]$$

$$= e^{-3.333t}\left[\underbrace{(a_1 + a_2)}_{A_1}\cos(11.542t) + \underbrace{i(a_1 - a_2)}_{A_2}\sin(11.542t)\right] \Rightarrow$$

$$\boxed{x = e^{-3.333t}\left[A_1\cos(11.542t) + A_2\sin(11.542t)\right]}$$ پاسخ سیستم

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{2000}{15}} \simeq 11.547 \ \frac{rad}{s} \qquad \omega_n = 11.547 \ rad/s$$

$$c_c = 2m_{eq}\omega_n = 2\sqrt{k_{eq}\, m_{eq}} = 2\sqrt{2000 \times 15} = 346.410 \ \frac{N.S}{m} \qquad c_c = 346.410 \ \frac{N.S}{m}$$

$$\zeta = \frac{c}{c_c} = \frac{10}{346.410} = 0.0289 \qquad \zeta = 0.0289$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 11.547\sqrt{1-0.0289^2} = 11.542 \qquad \omega_d \simeq 11.542 \ \frac{rad}{s}$$

مجهولات A_1 و A_2 را با استفاده از شرایط اولیه محاسبه می‌کنیم

$$I.C. \begin{cases} x_{(0)} = 0 \\ \dot{x}_{(0)} = 10 \end{cases}$$

$$x = e^{-3.333t}\left[A_1\cos(11.542t) + A_2\sin(11.542t)\right] \xRightarrow{t=0}$$

$$x_{(0)} = 0 = e^{0}\left[A_1\cos(0) + A_2 \times \sin 0\right) \Rightarrow \boxed{A_1 = 0} \Rightarrow \boxed{x = A_2 e^{-3.333t}\sin(11.542t)}$$

$$\Rightarrow \dot{x} = -3.333 A_2 e^{-3.333t}\sin(11.542t) + 11.542 \times A_2 e^{-3.333t}\cos(11.542t) \Rightarrow$$

$$\dot{x}_{(0)} = 10 = 11.542 A_2 e^{0}\cos(0) \Rightarrow A_2 = \frac{10}{11.542} = 0.866 \qquad \boxed{A_2 \simeq 0.866}$$

$$x_{(t)} = x = 0.866\, e^{-3.333t}\sin(11.542t)$$