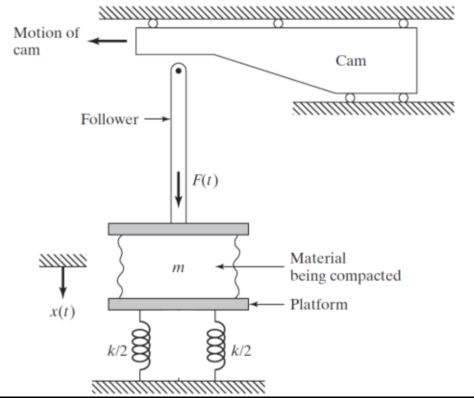
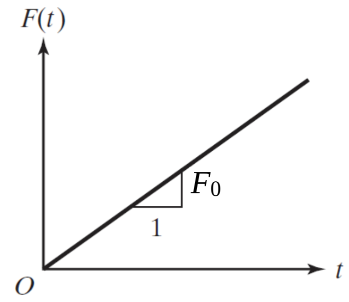
Final exam of Mechanical Vibration		University of Hormozgan
Name:	2021-2022-2	Dr. Mohammad Hosseini
Time: 120 min		Department of Mechanical Engineering

1. Determine the response of the compacting machine shown in figure when a linearly varying force is applied due to the motion of the cam.





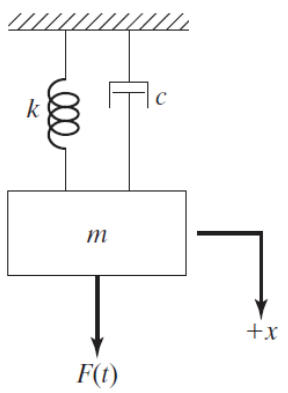
۱- پاسخ ماشین فشرده‌سازی نشان داده شده در شکل را هنگامی که یک نیروی متغیر خطی به دلیل حرکت بادامک اعمال می‌شود، به دست آورید.

پاسخ سیستم به ورودی ضربه واحد:

$$x(t) = \frac{1}{m\omega_n}\left(\sin(\omega_n t)\right)$$

۲۰ نمره

2. Consider a single-degree-of-freedom system subjected to a force $F(t)$, as indicated in figure. Find the steady-state response of the mass. Explain the effect of initial conditions on the steady-state response.



$$F(t) = F_0 e^{-\omega t}$$

$$m = 10kg, c = 20\frac{N.s}{m}$$

$$\omega = 2, F_0 = 5N,$$

$$k = 1000\frac{N}{m}$$

$$x(0) = 0, \dot{x}(0) = 1$$

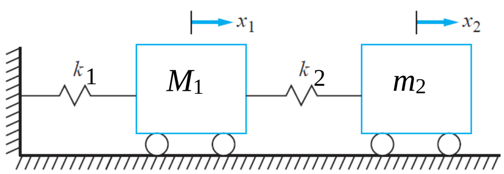
۲- سیستم یک درجه آزادی روبه‌رو را در نظر بگیرید که تحت نیروی $F(t)$ قرار دارد. این نیرو در شکل نشان داده شده است.

الف: پاسخ حالت پایدار سیستم را به دست آورید (۱۵ نمره).

ب: تأثیر شرایط اولیه را بر پاسخ حالت پایدار تشریح کنید (۱۵ نمره).

تمام جزئیات محاسبه پاسخ را ذکر کنید.

3. For the two degree of freedom system shown in Figure, Determine the system response as a function of time.



$$m_1 = m_2 = 5kg$$

$$k_1 = k_2 = 2000\frac{N}{m}$$

$$x_1(0) = x_2(0) = 0m$$

$$\dot{x}_1(0) = 0$$

$$\dot{x}_2(0) = 0.3\frac{m}{s}$$

۳- فرکانس‌های طبیعی و شکل مودهای سیستم نشان داده شده در شکل را به دست آورید. پاسخ سیستم را به شرایط اولیه داده شده محاسبه کنید.

استفاده از فرمول‌های آماده به هیچ عنوان قابل قبول نیست.

۵۰ نمره

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پاسخ سوال 1:

$g(t) = \frac{1}{m\omega_n} \sin\omega_n t$, $F(t) = F_0 t$

$$x(t) = \int_0^t F(\tau) g(t-\tau)\, d\tau = \int_0^t (F_0 \tau)\left(\frac{1}{m\omega_n} \sin(\omega_n (t-\tau))\right) d\tau \Rightarrow$$

$$\boxed{x(t) = \frac{F_0}{k}\left(t - \frac{1}{\omega_n} \sin\omega_n t\right) = \frac{F_0}{\omega_n k}\left(\omega_n t - \sin\omega_n t\right)}$$

جواب سوال 2:

پاسخ حالت ماندگار سیستم همان حل خصوصی معادله است: $m\ddot{x} + c\dot{x} + kx = F_0 e^{-\omega t}$

$$x_{p(t)} = B e^{-\omega t} \Rightarrow \dot{x}_{p(t)} = -\omega B e^{-\omega t} \Rightarrow \dot{x}_{p(t)} = \omega^2 B e^{-\omega t} \Rightarrow$$

$$B\left[m\omega^2 - c\omega + k\right] = F_0 \Rightarrow B = \frac{F_0}{m\omega^2 - c\omega + k} \Rightarrow \boxed{x_{p(t)} = \frac{F_0}{m\omega^2 - c\omega + k} e^{-\omega t}}$$

$$x_{p(t)} = \frac{5}{10 \times 2^2 - 20 \times 2 + 1000} e^{-2t} \Rightarrow \boxed{x_{p(t)} = 0.005 e^{-2t}}$$

★ شرایط اولیه هیچ تأثیری بر پاسخ حالت پایدار سیستم ندارد.

★ پاسخ حالت ماندگار سیستم همان حل خصوصی معادله است.

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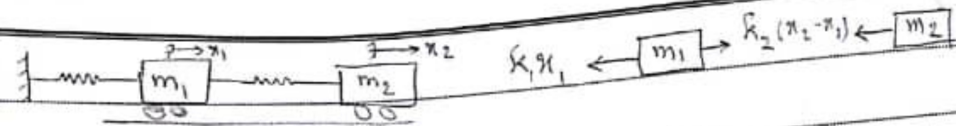
1, ③ سؤال



$$\begin{cases} \sum F_{x_1} = m_1\ddot{x}_1 \Rightarrow -k_1x_1 + k_2(x_2-x_1) = m_1\ddot{x}_1 \\ \sum F_{x_2} = m_2\ddot{x}_2 \Rightarrow -k_2(x_2-x_1) = m_2\ddot{x}_2 \end{cases} \Rightarrow$$

$$\begin{cases} m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0 \\ m_2\ddot{x} + k_2x_2 - k_2x_1 = 0 \end{cases} \quad ① \text{ and } \begin{cases} x_1 = X_1\sin(\omega t+\phi) \\ x_2 = X_2\sin(\omega t+\phi) \end{cases} ② \Rightarrow$$

Differetial Equation of motion

$$\begin{cases} \dot{x}_1 = \omega X_1\cos(\omega t+\phi) \\ \dot{x}_2 = \omega X_1\cos(\omega t+\phi) \end{cases} ③ \Rightarrow \begin{cases} \ddot{x}_1 = -\omega^2X_1\sin(\omega t+\phi) \\ \ddot{x}_2 = -\omega^2X_2\sin(\omega t+\phi) \end{cases} ④$$

$$②, ④ \text{ into } ① \Rightarrow \begin{cases} -m_1\omega^2X_1\sin(\omega t+\phi) + (k_1+k_2)X_1\sin(\omega t+\phi) - k_2X_2(\sin(\omega t+\phi)) = 0 \\ -m_2\omega^2X_2\sin(\omega t+\phi) + k_2X_2\sin(\omega t+\phi) - k_2X_1\sin(\omega t+\phi) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (k_1+k_2-m_1\omega^2)X_1\sin(\omega t+\phi) - k_2X_2\sin(\omega t+\phi) = 0 \\ (k_2-m_2\omega^2)X_2\sin(\omega t+\phi) - k_2X_1\sin(\omega t+\phi) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} (k_1+k_2-m_1\omega^2)X_1 - k_2X_2 = 0 \\ (k_2-m_2\omega^2)X_2 - k_2X_1 = 0 \end{cases} ⑤ \Rightarrow$$

دو معادله، دو مجهول

$$\begin{vmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{vmatrix} = 0 \quad ⑥ \Rightarrow$$

$$(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2) - k_2^2 = 0 \Rightarrow$$

$$m_1m_2\omega^4 - (m_1k_2+m_2(k_1+k_2))\omega^2 + k_1k_2 = 0 \quad ⑦ \begin{cases} a = m_1m_2 \\ b = m_1k_2+m_2(k_1+k_2) \\ c = k_1k_2 \end{cases}$$

$$a\omega^4 + b\omega^2 + c = 0, \ \Omega = \omega^2 \Rightarrow a\Omega^2 + b\Omega + c = 0$$

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$$\lambda = \omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+[m_1k_2 + m_2(k_1+k_2)] \pm \sqrt{[m_1k_2+m_2(k_1+k_2)]^2 - 4m_1m_2k_1k_2}}{2m_1m_2} \quad (8)$$

(8)

$$\omega = \omega_1 \Rightarrow r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{k_1+k_2-m_1\omega_1^2}{k_2} = \frac{k_2}{k_2 - m_2\omega_1^2} \quad (9) \Rightarrow$$

$$\begin{cases} x_{11} = X_1^{(1)} \sin(\omega_1 t + \phi_1) \\ x_{21} = X_2^{(1)} \sin(\omega_1 t + \phi_1) \end{cases} \Rightarrow \begin{cases} X_1^{(1)} \sin(\omega_1 t + \phi_1) = x_1^{(1)} \\ x_1^{(2)} = r_1 X_1^{(1)} \sin(\omega_1 t + \phi_1) \end{cases} \quad (10)$$

مود اول

(8)

$$\omega = \omega_2 \Rightarrow r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{k_1+k_2-m_1\omega_2^2}{k_2} = \frac{k_2}{k_2-m_2\omega_2^2} \quad (11)$$

$$\Rightarrow \begin{cases} x_{12} = X_1^{(2)} \sin(\omega_2 t + \phi_2) \\ x_{22} = X_2^{(2)} \sin(\omega_2 t + \phi_2) \end{cases} \Rightarrow \begin{cases} x_{12} = X_1^{(2)} \sin(\omega_2 t + \phi_2) \\ x_{22} = r_2 X_1^{(2)} \sin(\omega_2 t + \phi_2) \end{cases} \quad (12)$$

مود دوم

$$\Rightarrow \begin{cases} x_1(t) = x_{11} + x_{12} = X_1^{(1)} \sin(\omega_1 t + \phi_1) + X_1^{(2)} \sin(\omega_2 t + \phi_2) \\ x_2(t) = x_{21} + x_{22} = r_1 X_1^{(1)} \sin(\omega_1 t + \phi_1) + r_2 X_1^{(2)} \sin(\omega_2 t + \phi_2) \end{cases} \quad (13)$$

complete solution $\leftarrow x_2 , x_1$

$$\begin{cases} \dot{x}_1(t) = \omega_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + \omega_2 X_1^{(2)} \cos(\omega_1 t + \phi_2) \\ \dot{x}_2(t) = r_1 \omega_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2 \omega_2 X_1^{(2)} \cos(\omega_2 t + \phi_2) \end{cases} \quad (14)$$

Initial Conditions (I.C.) $\begin{cases} x_1(t=0) = x_{10} \\ x_2(t=0) = x_{20} \end{cases} , \begin{cases} \dot{x}_1(t=0) = \dot{x}_{10} \\ \dot{x}_2(t=0) = \dot{x}_{20} \end{cases}$ (15)

(15) into (13) and (14)

$$\begin{cases} x_{10} = X_1^{(1)} \sin\phi_1 + X_1^{(2)} \sin\phi_2 \\ x_{20} = r_1 X_1^{(1)} \sin\phi_1 + r_2 X_1^{(2)} \sin\phi_2 \\ \dot{x}_{10} = \omega_1 X_1^{(1)} \cos\phi_1 + \omega_2 X_1^{(2)} \cos\phi_2 \\ \dot{x}_{20} = r_1 \omega_1 X_1^{(1)} \cos\phi_1 + r_2 \omega_2 X_1^{(2)} \cos\phi_2 \end{cases} \quad (16)$$

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$a = m_1 m_2 = 5 \times 5 = 25$

$b = -[m_1 k_2 + m_2(k_1 + k_2)] = -[5(2000) + 5(2000+2000)] = -30000$

$c = k_1 k_2 = 2000 \times 2000 = 4000000$ 17

$$\omega^2 = \frac{30000 \pm \sqrt{30000^2 - 4 \times 25 \times 4 \times 10^6}}{2 \times 25} \Rightarrow \begin{cases} \omega_1^2 = 1047.21 \\ \omega_2^2 = 152.786 \end{cases} \Rightarrow \begin{cases} \omega_1 = 32.361 \ \frac{rad}{s} \\ \omega_2 = 12.361 \ \frac{rad}{s} \end{cases}$$

(9) $\Rightarrow r_1 = \dfrac{4000 - 1047.21 \times 5}{2000} = -0.618$, $r_2 = \dfrac{4000 - 152.786 \times 5}{2000} = 1.6181$ 18

16
$$\begin{cases} X_1^{(1)} \sin \phi_1 + X_1^{(2)} \sin \phi_2 = 0 & (a) \\ -0.618 X_1^{(1)} \sin \phi_1 + 1.6181 \sin \phi_2 = 0 & (b) \\ 32.361 X_1^{(1)} \cos \phi_1 + 12.361 X_1^{(2)} \cos \phi_2 = 0 & (c) \\ -20 X_1^{(1)} \cos \phi_1 + 20 X_1^{(2)} \cos \phi_2 = 0.3 & (d) \end{cases}$$ 19

(a) $\Rightarrow \sin \phi_2 = -\dfrac{X_1^{(1)}}{X_1^{(2)}} \sin \phi_1$ (e) (e) into (b) $\Rightarrow \sin \phi_1 = 0 \Rightarrow \phi_1 = 0$

$\Rightarrow \phi_2 = 0 \Rightarrow \boxed{\phi_1 = \phi_2 = 0}$

$$\begin{matrix} c \\ d \end{matrix} \begin{cases} 32.361 X_1^{(1)} + 12.361 X_1^{(2)} = 0 \\ -X_1^{(1)} + X_1^{(2)} = \dfrac{0.3}{20} \end{cases} \Rightarrow \begin{cases} X_1^{(1)} = -0.004145 \Rightarrow X_2^{(1)} = 0.002562 \\ X_1^{(2)} = 0.010854 \Rightarrow X_2^{(2)} = 0.017562 \end{cases}$$

$$\begin{cases} x_1(t) = -0.0004145 \sin(32.361t) + 0.010854 \sin(12.361t) \\ x_2(t) = 0.002526 \sin(32.361t) + 0.017629 \sin(12.361t) \end{cases}$$ 20