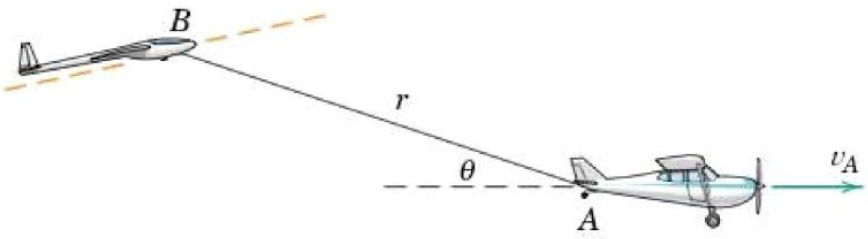
Dynamics Meriam 6ed, 2-206

Wednesday, November 24, 2021 11:42 AM

Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitude. If the tow cable has a length r = 60 m and θ is increasing at the constant rate of 5 degrees per second, determine the magnitudes of the velocity **v** and acceleration **a** of the glider for the instant when $\theta = 15°$.

$\vec{a}_A = 0$

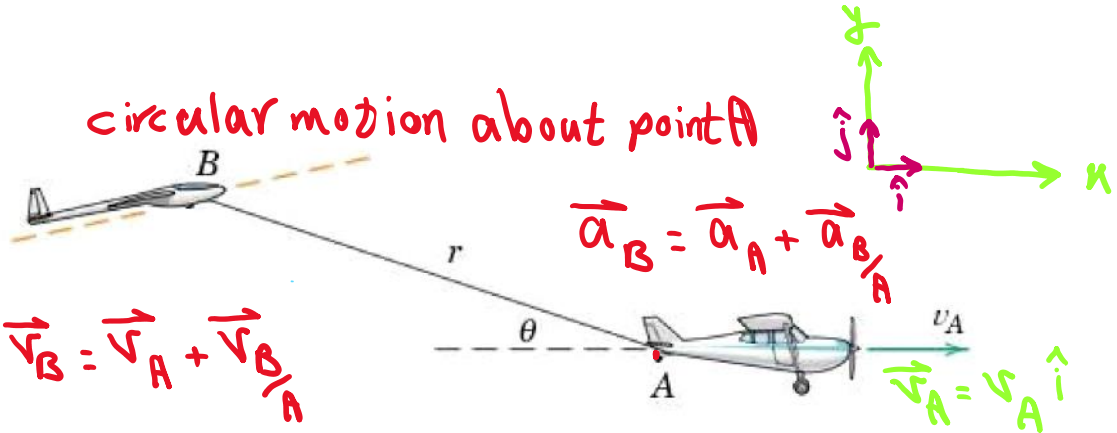


circular motion about point A

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{v}_A = v_A \hat{i}$



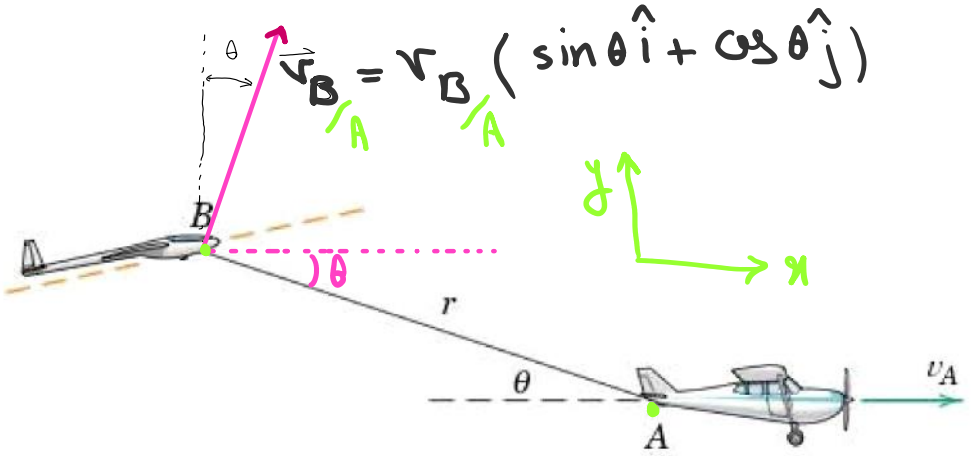
$v_A = 200 \frac{km}{h} = 200 \frac{1000}{3600}$

$r = 60$ m

$\dot{\theta} = 5 \frac{deg}{s} = 5 \frac{\pi}{180} \frac{rad}{s}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$, $\vec{v}_A = v_A \hat{i}$ (1)

$\vec{v}_{B/A} = v_{B/A}(\sin\theta \hat{i} + \cos\theta \hat{j})$



circular motion for B: $\vec{v}_B = r\dot{\theta}(\sin\theta \hat{i} + \cos\theta \hat{i})$ (2)

circular motion for B/A: $\vec{v}_{B/A} = r\dot{\theta}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$ (2)

(1), (2) $\Rightarrow \vec{v}_B = \underbrace{v_A\hat{i}}_{\vec{v}_A} + \underbrace{(r\dot{\theta}\sin\theta\,\hat{i} + r\dot{\theta}\cos\theta\,\hat{j})}_{\vec{v}_{B/A}} \Rightarrow$

$$\boxed{\vec{v}_B = (v_A + r\dot{\theta}\sin\theta)\hat{i} + r\dot{\theta}\cos\theta\,\hat{j}} \quad (3)$$

$$\left.\begin{cases} r = 60\ m \\ \dot{\theta} = 5\times\frac{\pi}{180}\ \frac{rad}{s} \end{cases}, \quad \theta = 15\right\} \Rightarrow$$

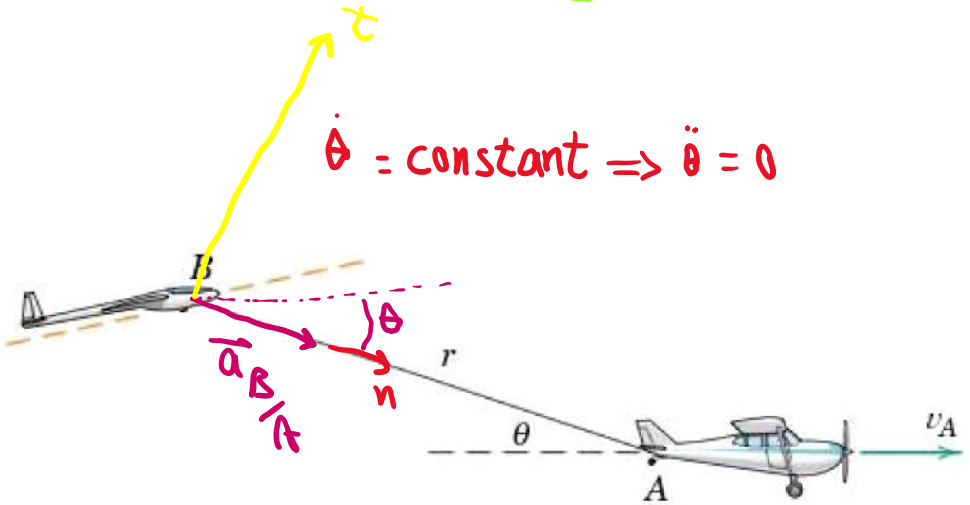
$$\vec{v}_B = \left(200\times\frac{1000}{3600} + 6\times\left(5\times\frac{\pi}{180}\right)\sin 15\right)\hat{i} + 60\times\left(5\times\frac{\pi}{180}\right)\cos 15\,\hat{j} \Rightarrow$$

If $\theta = 15° \Rightarrow \vec{v}_B = 56.910\,\hat{i} + 5.058\,\hat{j}\ \left(\frac{m}{s}\right)$

$$v_B = |\vec{v}_B| = \sqrt{(56.910)^2 + (5.058)^2} = 57.134\ \frac{m}{s}$$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\dot{\theta}$ = constant $\Rightarrow \ddot{\theta} = 0$



$\vec{a}_A = 0 \Rightarrow \vec{a}_B = 0 + \vec{a}_{B/A} \Rightarrow \vec{a}_B = \vec{a}_{B/A}$

$\vec{a}_{B/A} = r\dot{\theta}^2(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) \Rightarrow$

$$\boxed{\vec{a}_B = r\dot{\theta}^2(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})} \quad (3)$$

If $\theta = 15 \Rightarrow \vec{a}_B = \vec{a}_{B/A} = 60 \times (5 \times \frac{\pi}{180})^2 (\cos 15\, \hat{i} - \sin 15\, \hat{j}) \Rightarrow$

$$\vec{a}_B = \vec{a}_{B/A} = 0.441\, \hat{i} - 0.118\, \hat{j} \quad (\frac{m}{s^2})$$

$$a_B = |\vec{a}_B| = \sqrt{(0.441)^2 + (0.118)^2} \simeq 0.457 \ \frac{m}{s^2}$$